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ANALYSIS OF AERODYNAMIC DRAWING OF A THIN NONISOTHERMAL JET OF A VISCOELASTIC FLUID

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The essence of aerodynamic drawing of a jet (ADJ) lies in the fact that the drawing force is an aerodynamic frictional force between the jet and the air $F_{ar}(x)$. The author [1] proposed a mathematical ADJ model that allows one to calculate the jet parameters under various conditions of its motion.

The goal of the present paper is to study the jet parameters after drawing (when they remain almost unchanged for $x = L$) versus various conditions of its motion, namely, to find the final velocity $V = v(L)$ or diameter $D = d(L)$ and temperature $T(L) = T_f$ of the jet versus the conditions of its motion.

The ADJ simulation results can find applications in the development of the technology of aerodynamic formation of chemical fibers (AFF). In AFF, the drawing force is a frictional force between the fiber and the air that appears as a result of using an injector which is placed on the path of fiber motion and forms an air flow along the fiber being formed. In production of nonwoven materials from a polymer melt, this method makes it possible to produce the fiber and the final product in a single technological step. One should mention that the final diameter D and temperature T_f of the fiber determine to a considerable extent the quality of the nonwoven material obtained by self-adhesive fiber connection. Some technological and physical AFF problems were considered by Genis et al. [2–4]. Some mathematical AFF models are known [5, 6].

1. Basic Equations and Boundary Conditions. For a more adequate description of ADJ, a modified mathematical model that is more perfect in comparison with that in [1] is proposed. According to [7, 8], a more general equation of jet motion that incorporates the viscoelasticity of a fluid whose viscosity is representable as the function $\mu(T, dv/dx)$ [7] was derived based on the balance of forces acting on the jet during motion:

$$v'' + \{[\ln(\mu/v)]' + [A_{sf}v^{-0.5} - \rho v]\}v'/\mu + \{\rho g - \operatorname{sign}(\Delta v)A_{ar}\Delta v^{2-\xi}v^{0.5(\xi+1)}\}/\mu = 0. \qquad (1.1)$$

Here and below, $v' = dv/dx$; the prime denotes differentiation with respect to x, x is the coordinate along the jet propagation, g is the acceleration of gravity, ρ is the density of the fluid, $c_f = a_c \mathrm{Re}^{-\xi}$ is the coefficient of aerodynamic friction, ρ_0 is the density of the medium, $\Delta v = v(x) - u(x)$ is the difference between the velocities v and u of the jet and the medium, respectively, $\mathrm{Re} = 2R|\Delta v|/\nu_0$ is the Reynolds criterion, a_c and ξ are the constants, σ is the coefficient of surface tension, G is the flow rate of the fluid, ν_0 is the kinematic viscosity of the medium, $R(x)$ is the current jet radius, $A_{ar} = a_c\rho_0(2/\nu_0)^{-\xi}(\pi\rho/G)^{0.5(1+\xi)}$, and $A_{sf} = (\sigma/2)\sqrt{\pi\rho/G}$.

The one-dimensional equation (1.1) was obtained under the assumption that the distribution of the axial velocity over the transverse cross section of the jet is uniform. The basic assumptions of the theory of motion of a thin jet are given by Chang [7] and Zyabitskii [8].

For fluid viscosity, according to [7, 9], the function $\mu(T, dv/dx)$ is taken in the form

$$\mu = 3\mu_0 \exp[\beta(1/T - 1/T_0)]\{m_0 + m_1(v')^{q-1}\}, \qquad (1.2)$$

where μ_0 is the longitudinal viscosity at temperature T_0, $\beta = E/R_c$, E is the activation energy, R_c is the gas constant, and m_0, m_1, and q are the constants depending on the nature of the material.

One-dimensional equations were used to describe the heat exchange of the jet and the medium [1–8]. However, analysis has shown that for a thin jet, with radius $R \sim 5 \cdot 10^{-5}$ m, velocity $v \sim 100$ m/sec, and

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motion path $L \sim 1$ m, the uniform temperature distribution over the jet radius is not established. Therefore, for description of this process, one should apply a two-dimensional equation, because the temperature T has a strong effect on the viscosity μ (1.2) and on the jet motion. The heat-exchange equation is of the form [7]

$$C\rho(T)v\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\Big(\lambda(T)r\frac{\partial T}{\partial r}\Big). \tag{1.3}$$

Here $T(r,x)$ is the temperature of the jet, C is the specific heat, and λ is the coefficient of thermal conductivity. For $x = 0$, the initial conditions are as follows:

$$T(r,0) = T_1(r), \tag{1.4}$$

and the boundary conditions at the outer boundary of the jet $r = R(x)$ are

$$-\lambda\frac{\partial T(R,x)}{\partial r} = \alpha[T(R,x) - T_s(x)], \tag{1.5}$$

where T_s is the ambient temperature, $\alpha = 2\lambda_0 a_n \mathrm{Re}^\gamma/R$ is the heat-transfer coefficient calculated from the relation $\mathrm{Nu} = a_n\mathrm{Re}^\gamma$ [8], λ_0 is the thermal conductivity of the medium, and a_n and γ are constants. In the center of the jet, the symmetry condition is

$$\frac{\partial T(0,x)}{\partial r} = 0. \tag{1.6}$$

The coupled system (1.1)–(1.6) is a mathematical model of motion of a nonisothermal jet. The temperature distribution over the jet length affects its viscosity (1.2) and, according to (1.1), the velocity of the jet. The jet velocity affects, in turn, the temperature distribution through the heat-transfer coefficient (1.5). In view of this, system (1.1)–(1.6) was solved jointly. The ordinary differential equation (1.1) was solved by the fourth-order Runge–Kutta method, with a constant step. The presence of the moving boundary $R(x)$ creates additional difficulties in solving Eq. (1.3), and, hence, for transition from the unstationary $R(x)$ to stationary boundary, the Mises transform [10] was used, which introduces a new nondimensional variable η determined by the formula $\eta = r/R(x)$. To solve the nonlinear equation (1.3), the implicit scheme of the finite-difference method [10] was employed. The algebraic system of equations was linearized by the method of iterations, whose number did not exceed three in calculations.

Together with the algorithm of selection of boundary conditions [1], the mathematical ADJ model makes it possible to calculate the jet parameters under various conditions of its motion.

2. Identification of the Mathematical Model. For numerical ADJ simulation, we use the values of the parameters of the AFF technological process and also the experimental data from [2–4]; the viscoelastic fluid is a polypropylene melt. The numerical values for all the quantities used are given in [1].

For a numerical simulation of the jet motion, a mathematical model should correspond to a real process parametrically. For this purpose, the residue method [11] with respect to the experimental velocity $v(x)$ and temperature $T(x)$ distributions of the jet along the x motion, which are known from [2–4], was used. Identification is necessitated by the determination of all coefficients used in the mathematical ADJ model. The identification criterion was chosen in the form of the functional

$$I = \int_0^L [\varepsilon_v^2(x) + \varepsilon_T^2(x)]\,dx \to \min, \tag{2.1}$$

where $\varepsilon_v(x) = v_e(x) - v_r(x,\mathrm{k})$ and $\varepsilon_T(x) = T_e(x) - T_r(x,\mathrm{k})$ are the differences of the experimental and calculated values of the velocity and temperature distributions. The coefficients to be identified are presented as the vector $\mathrm{k} = \{\mu_0, \beta, m_0, m_1, q, a_n, \gamma, a_c, \xi\}$ whose initial values are taken from [2–4]. Satisfactory agreement of the calculational and experimental data, which corresponds to the fullfilment of relation (2.1), was obtained for $a_c = 1$ and $\xi = 0.55$, with other k components being unchanged. The change of only the components a_c and ξ in the vector k in the identification is caused by the fact that precisely these components determine c_f (1.1) and, therefore, affect to a considerable extent the drawing force $F_{ar}(x)$ [8], which mainly contributes to the dynamics of jet motion upon aerodynamic drawing.

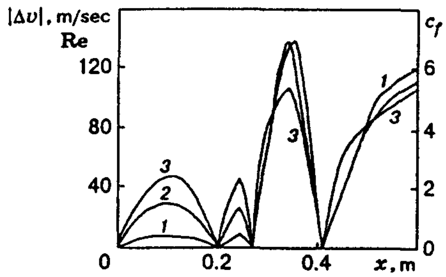

Fig. 1

To find the dependence $V(\mathbf{k})$ at the above-indicated initial values of the coefficients and successive variation of one of them, a numerical simulation was performed. The results of simulation and qualitative analysis of the dependence $V(\mathbf{k})$ coincide. As an illustration, we consider the inversely proportional dependence of the final jet velocity V on the fluid viscosity at the initial temperature, denoted as $V \sim 1/\mu_0$. An increase in μ_0 will lead to an increase in μ (1.2) and in the rheological force $F_{rh}(x)$. Following from the law of energy conservation, the work of the drawing force $F_{ar}(x)$ is spent on the work on jet drawing, proportional to the rheological force, and on the kinetic energy, proportional to the square of the filament velocity v^2. An increase in $F_{rh}(x)$, therefore, causes a decrease in V.

3. Analysis of Jet Motion. It is interesting to analyze the character of variation of the physical coefficients Re, c_f, and Nu on the path of jet propagation. The dependences Re(x), $c_f(x)$, and Nu(x), obtained by simulation, are presented in Fig. 1, where curves 1–3 correspond to $|\Delta v(x)|$, Re, and c_f, respectively. The dependence Nu(x) is of a form similar to Re(x). It is worth noting that in ADJ, Re is dependent on the relative velocity Δv of the jet and the medium, rather than the jet velocity v. The function $\Delta v(x)$ is sign-variable, and, therefore, its modulus enters Re. According to relations (1.1) and (1.5), precisely $\Delta v(x)$ and then Re determine the values of the coefficients c_f and Nu.

The dependence of the properties of the jet at the end of motion on the basic motion parameters $T_s(x)$, $u(x)$, and G, which we denote as the components of the vector $\mathbf{c}$, was obtained by simulation as well. The dependence $V(\mathbf{c})$ was found by the method of successive variation of the component $\mathbf{c}$, with constant values of the others. The results are given in Figs. 2 and 3. Curves 1–6 in Fig. 2 correspond to $V(G)$, $V(T_s)$, $V(L_s)$, $T_f(G)$, $T_f(T_s)$, and $T_f(L_s)$. The reference values of $T_s(x)$ and $u(x)$ are taken from [2–4].

In finding the dependence $V[T_s(x)]$, the temperature function of the medium was used in the form ($L_s = 0.2$ m)

$$T_s(x) = \begin{cases} T_s = \text{const} & \text{for} \quad x < L_s, \\ 300 \text{ K} & \text{for} \quad x > L_s, \end{cases}$$

where L_s is the duration of the stepwise function T_s along the path of motion, which was varied as well. As the temperature T_s of the medium rises, the mean temperature of the jet along the motion increases, which, in accordance with (1.2), decreases the value of viscosity μ and leads to an increase in V, for an unvaried drawing force. The dependence obtained is close to a linear one, which is approximated satisfactorily by the linear function $V = 74 + 0.18T_s$ (T_s in degrees C). Similar arguments explain the proportional dependence $V \sim L_s$ (see Fig. 2).

In the simulation, the function of air velocity (of the medium of jet motion) is as follows:

$$u_s(x) = \begin{cases} u(x) & \text{for} \quad x < 0.3 \text{ m}, \quad x > 0.4 \text{ m}, \\ u = \text{const} & \text{for} \quad 0.3 < x < 0.4 \text{ m} \end{cases}$$

(the experimental value of $u(x)$ is taken from [2–4]). An increase in u_s leads to an increase in the drawing force $F_{ar}(x)$ and, according to the law of energy conservation, gives rise to the increase in the final jet velocity V. The dependence $V(u_s)$ is close to a linear one, which is satisfactorily approximated by the linear function $V = 0.8u_s - 14$.

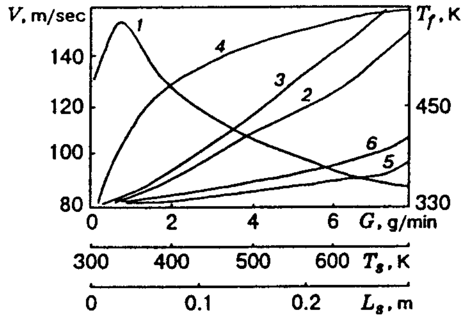

Fig. 2

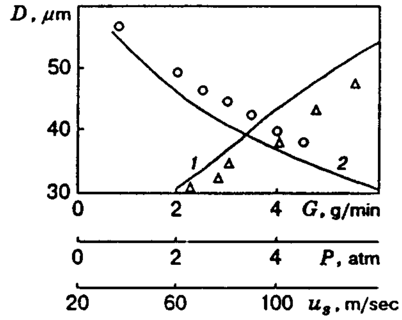

Fig. 3

The dependences $V(c)$ and $T_f(c)$ obtained are monotone functions, except for $V(G)$, which has a maximum for $G = 0.7$ (see Fig. 2). This maximum is accounted for by the fact that the initial radius of the jet $R(0)$ is proportional to G, and, therefore, for $G < 0.7$ the filament becomes rather thin and, as a consequence, its cooling for the motion time leads to a drastic increase in viscosity μ and to a decrease in jet velocity V. This is supported by calculations of the temperature distribution in the jet. For $G > 0.7$, the inertial force grows substantially with increasing G, thereby decreasing V for a constant drawing force.

For comparison, Fig. 3 shows the experimental data from [12] [points (▵) refer to $D(G)$ and points (○) refer to $D(P)$] and the calculational data (curves 1 and 2 refer to $D(G)$ and $D(u_s)$, respectively). The calculational dependence $D(u_s)$ has been obtained, which is compared with the experimental dependence on air pressure P in the injector. The comparison of these functions is possible owing to the linear dependence between the pressure and velocity of the air leaving the injector [13].

A satisfactory coincidence of the calculational results has been obtained, which demonstrates the adequacy of this mathematical model to a real phenomenon. The discrepancy between the calculational and experimental data can be explained by both the incorrectness of the mathematical model used, which was developed using some assumptions, and the incomplete information on the conditions of the experiments the results of which were reported in [12].

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